FLOW OF NON-NEWTONIAN FLUID IN A CHANNEL WITH A LARGE CAVITY

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The problem of fluid flow in a channel with a cavity is of interest for numerous applications. Many papers (see the review [1]) have been devoted to different variants of this problem for the case of a Newtonian fluid. The literature, however, contains hardly any papers dealing with corresponding flows of non-Newtonian fluids [2, 3]. Yet the problem of a flow of viscoplastic fluid in a gap with a depression is of interest for oil-drilling practice, since it models to some extent the process of cementation of wells with cavities in the walls.

1. As a model of a non-Newtonian fluid we selected a quasiplastic Williamson fluid [4], in which the effective viscosity η is given by the expression $\eta = \eta_\infty + \tau_0/(b + I)$, where η_∞ is the dynamic Newtonian viscosity at infinitely high shear velocities; τ_0 is an analog of the ultimate shear stress of a Bingham fluid; b is a parameter of the Williamson model; I is the square root of the second invariant of the strain rate tensor. When b = 0 the model represents a viscoplastic Bingham fluid; τ_0 = 0 corresponds to the case of a Newtonian fluid.

We write the dimensionless equations for plane steady flow in terms of the stream function ψ and vorticity Ω:

$$\mathrm{Re}\left(\frac{\partial\psi}{\partial x}\frac{\partial\Omega}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial\Omega}{\partial x}\right) + \Delta(\nu\Omega) - \Omega\Delta\nu = 4\frac{\partial^2\nu}{\partial x\partial y}\frac{\partial^2\psi}{\partial x\partial y} + \left(\frac{\partial^2\nu}{\partial x^2} - \frac{\partial^2\nu}{\partial y^2}\right)\left(\frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial y^2}\right),$$

$$\Delta\psi = -\Omega, \tag{1.1}$$

$$\nu = 1 + T\Bigg/\left\{B + \mathrm{Re}\left[4\left(\frac{\partial^2\psi}{\partial x\partial y}\right)^2 + \left(\frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial y^2}\right)^2\right]^{1/2}\right\}.$$

As units of length and velocity we select the channel half-width d and the mean velocity U, determined from the flow rate; $\nu = \eta/\eta_\infty$ is the dimensionless effective viscosity. Equations (1.1) contain three similarity criteria: $\mathrm{Re} = U\rho d/\eta_\infty$ is the Reynolds number, $T = \tau_0\rho d^2/\eta_\infty^2$ is an analog of the Hedström number for a Bingham fluid, and $B = b\rho d^2/\eta_\infty$ is the dimensionless parameter of the Williamson model.

The shape of the calculated region is shown in Figs. 1 and 2. As boundary conditions in the entrance and exit sections, situated sufficiently far from the cavity, the exact solution of Eq. (1.1), corresponding to plane-parallel flow [5], is used. The other boundaries are regarded as solid and the conditions for ψ and Ω on them are imposed in the usual way.

To solve the equations we used a difference scheme involving approximations of the first derivatives of the vorticity in (1.1) by one-sided differences directed "against the flow," and a central-difference approximation of the other derivatives. The system of difference equations was solved by Liebmann's iterative method with successive low relaxation.

2. We consider some of the results of the calculations. Of most interest are flows in long and deep cavities, which are often encountered in natural conditions. Hence, all the streamline maps referred to below relate to a cavity with dimensionless depth H = 20 and length L = 120.

A picture of a flow of Newtonian fluid is shown in Fig. 1a (Re = 1800). We consider a jet formed by sudden expansion of the cross section of the channel and impinging on the rear wall of the cavity, and also the region of the slower return flow near the solid boundaries of the cavity. The center of the eddy motion is much nearer the real wall, which blocks the path of the jet, and deflects the combined mass sharply into the cavity. As the calculations showed, a similar flow structure is found in square cavities, which is in good qualitative agreement with experimental data (see, e.g. [6]).

The nature of the motion of a non-Newtonian fluid depends considerably on the inhomogeneity of the spatial distribution of the effective viscosity, which depends on the velocity gradients and the parameters B, T, and Re.

The parameter B was chosen so that the considered fluid was similar in its properties to a Bingham fluid. We usually put B = 0.01 T; as test calculations showed, a further reduction of B has practically no effect on the flow structure.

With increase in the Hedström number T from zero to T = 225 at constant Re = 1800 (Fig. 1a, b) the formation of a secondary vortex and a stagnation region at the bottom of the cavity is accompanied by a reduction of the volume of the cavity occupied by the main vortex. The formation of a stagnation region at the bottom of the cavity is probably due to its great depth. Calculations made for a cavity with H = 9 and L = 60 (i.e., for H/L close to 20/120) reveal similar flow structures, but without a stagnation region.

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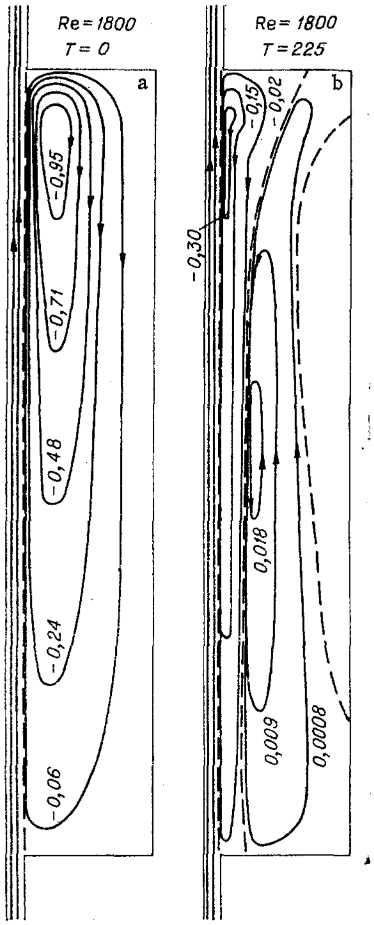

Fig. 1

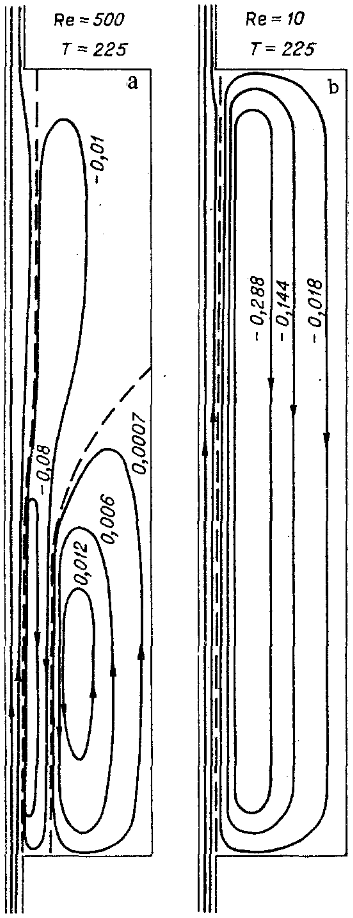

Fig. 2

Reduction of the Reynolds number with parameter T constant and fairly large (T = 225) can lead to a significant change in the flow region – this is the transition illustrated in Figs. 1b and 2a. The center of the main vortex is shifted towards the far edge of the cavity, its intensity in this case is greatly reduced, at the rear edge of the cavity a stagnation region is formed, and there is also some intrusion of the main through flow into the cavity to a depth of the order of the channel width.

Figure 2b shows the creeping flow regime observed at low Re and large T. A further decrease in Re leads, on one hand, to an increase in viscosity and, on the other, to a reduction of the contribution of the inertial terms to the equation of motion. At very low Re there is a considerable intrusion of flow in the middle of the cavity – this regime was considered in detail in [5].

An analysis of the calculated data shows that with Re constant the effect of increase in T on the flow structure is similar to some extent to the effect of reduction of Re with T constant.

We briefly recall the results relating to cavities of other dimensions. For instance, a series of calculations made for cavities of different depths (from H = 4 to H = 32) with length L = 120 showed that in the flow regime illustrated in Fig. 2a (Re = 1800, T = 225) the absolute depth of intrusion at the rear edge of the cavity increases a little with reduction of H. When H = 4 the through flow at the rear edge reaches the "bottom" of the cavity.

In square cavities the changes in flow structure are not so appreciable as in long cavities. The center of the main vortex here is always located at the rear edge of the cavity, and a flow with one or several vortices is observed (depending on the depth of the cavity).

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RHEOLOGICAL EQUATIONS OF STATE OF WEAKLY CONCENTRATED SUSPENSIONS OF DEFORMABLE ELLIPSOIDAL PARTICLES

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In [1], from the standpoint of a structural-continuous approach [2, 3], rheological equations of state are obtained for dilute suspensions of deformable ellipsoidal particles, having internal elasticity and viscosity, with a dispersion medium which is a Newtonian liquid. In the present article these results are generalized for larger concentrations. Taking account of the effect of the hydrodynamic interaction of suspended particles on the rheological behavior of a suspension is effected using the Simha method [4].

As in [1], we shall model the suspended particles by an ellipsoid, having an internal linear elasticity and a linear viscosity (a Voigt body), changing its dimensions during the process of interaction with the dispersion medium, but conserving its volume and retaining the form of an ellipsoid of revolution. To set up the rheological equations of state of the suspensions under consideration using a structural-continuous approach, it is necessary to determine the perturbations introduced into the inhomogeneous flow of a dispersion medium by a suspended particle; here, to take account of the hydrodynamic interaction of the suspended particles, the boundary conditions "at infinity," in accordance with [4], must be referred to the surface of a sphere screening the particle; the sphere has a center coinciding with the center of the particle, and a radius $R = (ab^2/\Phi)^{1/3}$, where $2a$ and b are the length of the axis of rotation and the equatorial radius of the particle, respectively; Φ is the volumetric concentration of suspended particles.

We shall seek the solution of the hydrodynamic problem in the Stokes approximation by the method of successive approximations [5]. As a first approximation we take the solution obtained in [1] for the case where an unbounded dispersion flows around the particle, but where the boundary conditions "at infinity" are referred to the surface of a sphere, whose radius considerably exceeds the effective radius of the particle. In a movable system of coordinates x_i, with its origin at the center of the particle and axes coinciding in direction with the directions of the axes of an ellipsoidal particle, this solution has the form

$$\begin{gathered} u_i = u_{0i} + \frac{\partial}{\partial x_i}(D_j\chi_j) - \varepsilon_{ijk}K_j\frac{\partial \chi_j}{\partial x_k} + c_{jk}x_j\frac{\partial^2\Omega}{\partial x_i \partial x_k} - \\ - c_{ij}\frac{\partial \Omega}{\partial x_j} - \frac{4}{3R^3}(c_{ki} - c_{ik})x_k + \frac{4x_i\psi}{R^5} + \frac{5(R^2 - r^2)}{R^5}\frac{\partial\psi}{\partial x_i}, \\ p = p_0 + 2\mu c_{ij}\frac{\partial^2\Omega}{\partial x_i \partial x_j}, \end{gathered} \tag{1}$$

where u_i is the velocity; p is the pressure; u_{0i}, p_0 are the velocity and the pressure of the unperturbed flow; r is the modulus of the radius-vector; μ is the dynamic viscosity coefficient of the dispersion medium; Ω, χ_j, D_j, K_j are values determined in [6]; c_{ij} are values determined in [1]; $\psi = c_{ij}x_ix_j$; ϵ_{ijk} is a skew-symmetric Kronecker symbol.

The first approximation (1) does not satisfy the boundary conditions at the surface of the particle; here, the divergences do not exceed values of the order of $O(R^{-3})$.

We obtain the second approximation of the problem under consideration, adding to (1) a partial solution of the problem, satisfying the following boundary conditions:

$$u_i|_w = \frac{4}{3R^3}(c_{ki} - c_{ik})x_k - \frac{5}{R^5}\frac{\partial\psi}{\partial x_i},$$
$$u_i \to 0 \quad \text{for} \quad r \to \infty,$$

where $u_i|_w$ is the velocity at the surface of the particle. This partial solution has the form

$$u_i = \frac{\partial}{\partial x_i}(Q_j\chi_j) - \varepsilon_{ijk}H_j\frac{\partial\chi_j}{\partial x_k} + B_{jk}x_j\frac{\partial^2\Omega}{\partial x_i\partial x_k} - B_{ij}\frac{\partial\Omega}{\partial x_j}, \quad p = 2\mu B_{ij}\frac{\partial^2\Omega}{\partial x_i\partial x_j}, \tag{2}$$

where

$$Q_1 = -\frac{3d_{32}}{4R^3b^2\alpha_0'\beta_0'};$$

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